Problem 2.18

Find the probability currrent, J (Problem 1.14) for the free particle wave function Equation 2.95. Which direction does the probability flow?

["Equation 2.95" should be enclosed in parentheses as "Problem 1.14" is.]

Solution

The wave function in Equation 2.95 is

$$\Psi(x,t) = A\exp\left(ikx - \frac{i\hbar k^2}{2m}t\right). \tag{2.95}$$

According to Problem 1.14, the probability current is

$$\begin{aligned}
J(x,t) &= \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right) \\
&= \frac{i\hbar}{2m}\left[A\exp\left(ikx - \frac{i\hbar k^2}{2m}t\right)\frac{\partial}{\partial x}A^*\exp\left(-ikx + \frac{i\hbar k^2}{2m}t\right)\right. \\
&\qquad\left. -A^*\exp\left(-ikx + \frac{i\hbar k^2}{2m}t\right)\frac{\partial}{\partial x}A\exp\left(ikx - \frac{i\hbar k^2}{2m}t\right)\right] \\
&= \frac{i\hbar}{2m}\left[A\exp\left(ikx - \frac{i\hbar k^2}{2m}t\right)A^*\exp\left(-ikx + \frac{i\hbar k^2}{2m}t\right)(-ik)\right. \\
&\qquad\left. -A^*\exp\left(-ikx + \frac{i\hbar k^2}{2m}t\right)A\exp\left(ikx - \frac{i\hbar k^2}{2m}t\right)(ik)\right] \\
&= \frac{i\hbar}{2m}\left[|A|^2(-ik) - |A|^2(ik)\right] \\
&= \frac{i\hbar}{2m}\left(-2ik|A|^2\right) \\
&= \frac{\hbar k}{m}|A|^2,
\end{aligned}$$

a constant. If $k > 0$, then $J(x,t)$ is positive, meaning probability is flowing in the positive x-direction. If $k < 0$, then $J(x,t)$ is negative, meaning probability is flowing in the negative x-direction.